Determination of friction factors for ultrafiltration applied to radioactive waste

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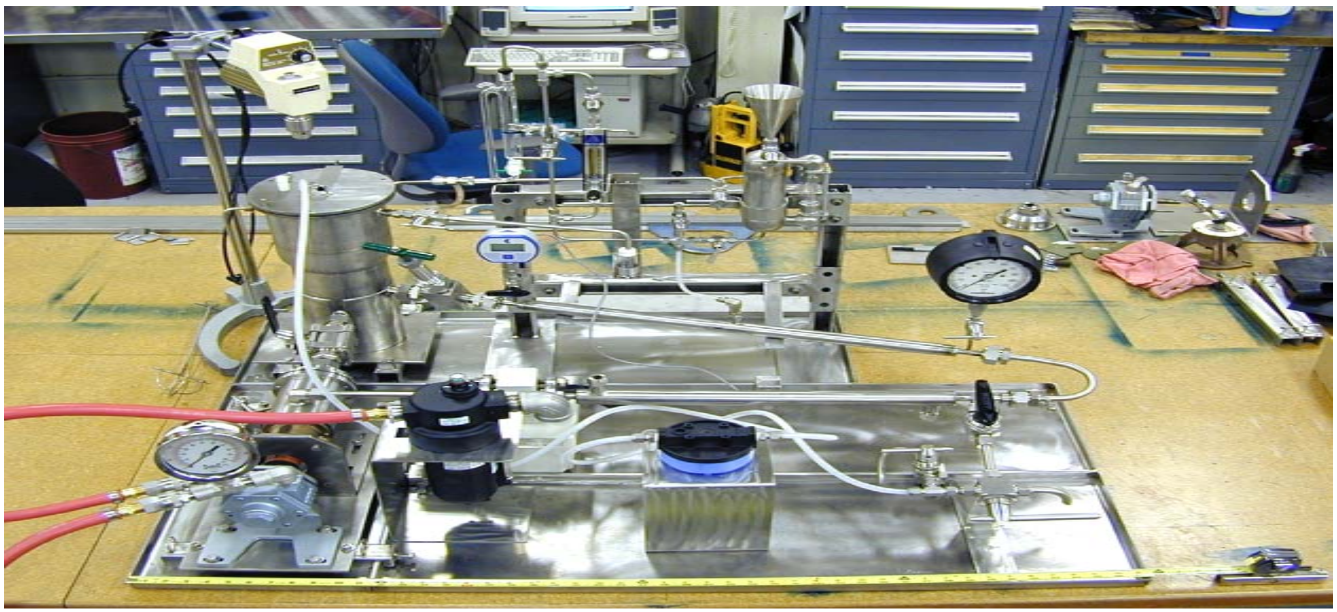

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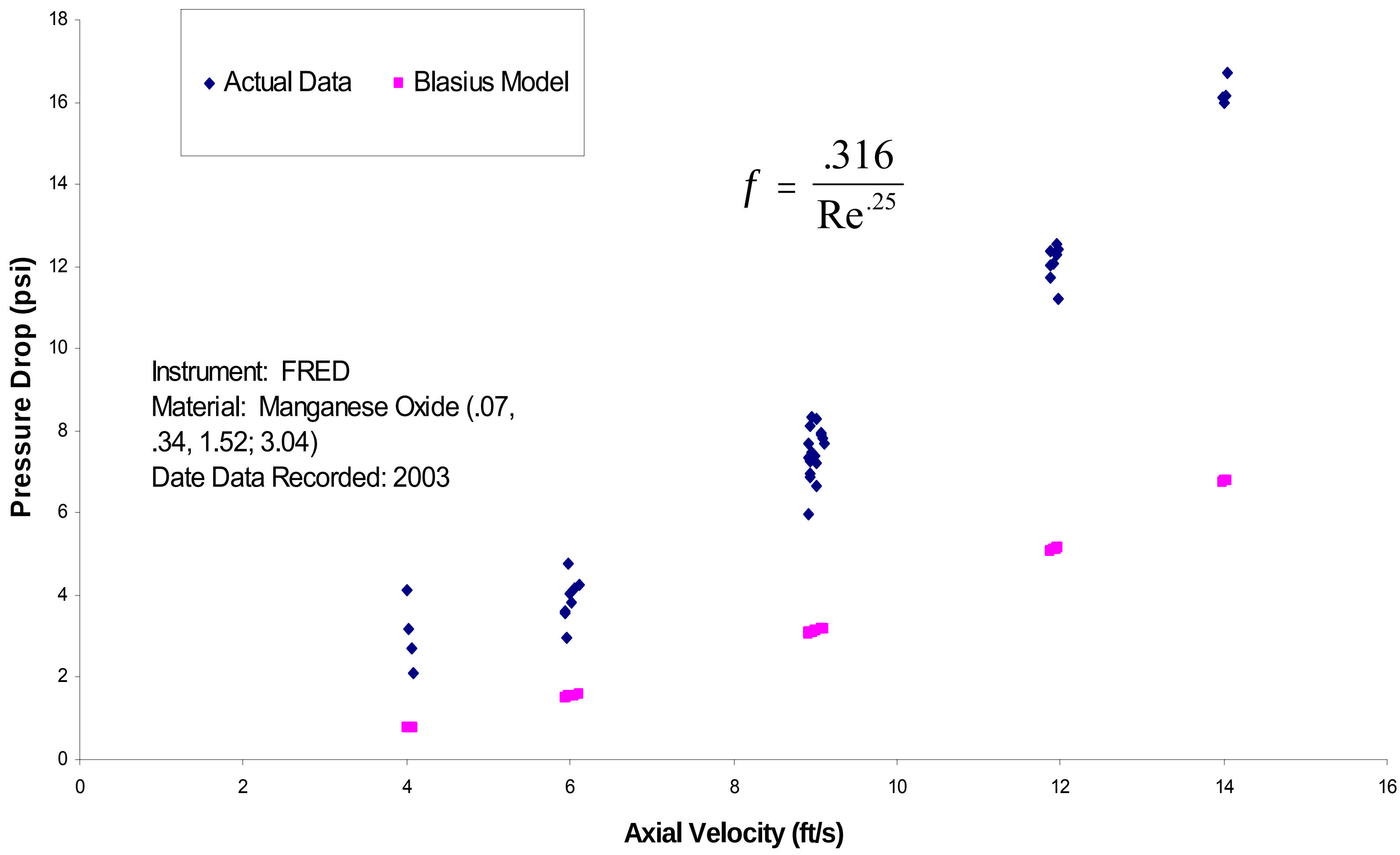
Outline

- friction factor model based on Blasius model
- friction factor model based on Prandtl's Mixing Length Theory
- friction factor model based on approximate momentum integral and methodology of measuring velocity profiles

Motivation
(Blasius Model)



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Theory
(Darcy-Weisbach Equation)

$$\frac{\Delta p(144)}{\gamma_s} = f\,\frac{L}{D}\,\frac{v^2}{2g}$$

Where:

Δp	is pressure drop
f	is friction factor
L	is length of porous section of tube
D	is the inner diameter
V	is axial velocity
γ_s	is specific weight

How do we define *f* for turbulent conditions?

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Theory
(Prandtl's Mixing Length Theory)

$$\tau_w = \rho k^2 y^2 \left(\frac{dv}{dy} \right)^2$$

$$dv = \frac{\sqrt{\tau / \rho}}{k} \frac{dy}{y}$$

$$\frac{v}{v^*} = \frac{1}{k} \ln y + C$$

$$\frac{v}{v^*} = \frac{1}{k} \ln(\mathrm{Re}) + B$$

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Theory
(Prandtl's Mixing Length Theory)

$$\frac{u}{u^*} = 5.75 \log \mathrm{Re} + B$$

$$\frac{u^*}{V} = \sqrt{\frac{f}{8}}$$

$$\sqrt{\frac{8}{f}} = 5.75 \log(\mathrm{Re}) + B$$

$$\frac{1}{\sqrt{f}} = a \log(\mathrm{Re}) + B$$

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Theory
(B Models)

- B= f(V, D, A, L, Q_p, ΔP, C_b, Q_s)
- B=f(Re, Cp, Q*, L/D, A/D^2)

$$B = f\left(C_p , \frac{L}{D} \right)$$

$$B = a(C_p)^b$$

Where
Δp is pressure drop
L is length of porous section of tube
D is the inner diameter
C_b is bulk phase concentration of solids
Q_p is permeate rates

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Theory

$$C_p = \frac{\Delta p 144}{\rho V^2}$$

$$\frac{\Delta p 144}{\gamma_s} = f \frac{L}{D} \frac{V^2}{2g}$$

$$C_p = \frac{f}{2} \frac{L}{D}$$

$$\frac{1}{\sqrt{f}} = .25 \log(\mathrm{Re}) + a \left(\frac{f}{2} \frac{L}{D} \right)^b$$

Method
(Finding f(i))

$$f_i = \frac{\Delta p_i \, 144}{\frac{1}{2} \rho \frac{L}{D} {v_i}^2}$$

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Method

(Finding B(i))

$$B_i = \frac{1}{\sqrt{f_i}} - a\log(\mathrm{Re}_i)$$

Method

(Models for B)

$$B = a\left(C_p\right)^b$$

$$\log\left(B\right) = \log\left(a\right) + b\log\left(C_p\right)$$

$$Y = AX$$

$$A = \begin{bmatrix} 1 & \log\left(C_{p_1}\right) \\ \vdots & \vdots \\ 1 & \log\left(C_{p_N}\right) \end{bmatrix}$$

$$X = \begin{bmatrix} \log a \\ b \end{bmatrix}$$

Parameters within **X** solved using least squares

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Research Questions

- Compare Blasius Model with developed model
- What physical mechanisms affect B?
- Is B affected by physical scale?
- Is porosity significant?

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Results
(Data Utilized)

Apparatus	Material	L (feet)	D (feet)	L/D	Viscosity (ft^2/s)	Density (lbs/ft^3)	R^2 (B)	R^2 (dP)
FRED	MN-Oxide	10	0.05208	192	8.05E-06	2.4192	0.9946	0.9989
	Water 30°C	10	0.05208	192	8.90E-06	2.4192	0.9853	0.9994
	Water 50°C	10	0.05208	192	5.70E-06	2.4192	0.9992	0.9996
CUF 01	$SrCO_3$ (exp. 4) 24°C	2	0.03125	64	9.72E-06	2.4192	0.9928	0.9989
	$SrCO_3$ (exp. 5) 26°C	2	0.03125	64	9.28E-06	2.4192	0.9979	0.9995
	$SrCO_3$ (exp. 6) 25°C	2	0.03125	64	9.50E-06	2.4192	0.9994	0.9997
	$SrCO_3$ (exp. 7) 25°C	2	0.03125	64	9.50E-06	2.4192	0.9984	0.9997
	$SrCO_3$ (exp. 8) 25°C	2	0.03125	64	9.50E-06	2.4192	0.9975	0.9996
	Water 25°C	2	0.03125	64	9.50E-06	2.4192	0.993	0.9933
CUF 07	Low Solids	2	0.04167	48	1.28E-05	2.4192	0.9672	0.9269

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Results
(Model Parameters)

					95% Confidence Interval		
Apparatus	**Material**	**log(a)**	**b**	**log(a) (low)**	**log(a) (high)**	**b (low)**	**b (high)**
FRED	MN-Oxide	0.9911	-0.6943	0.9775	1.0046	-0.7119	-0.6768
	Water 30^{o}C	0.9483	-0.6236	0.9169	0.9798	-0.6791	-0.5681
	Water 50^{o}C	0.9449	-0.6255	0.9375	0.9523	-0.6407	-0.6103
CUF 01	$SrCO_3$ (exp. 4) 24^{o}C	0.7057	-0.8742	0.6981	0.7133	-0.8923	-0.856
	$SrCO_3$ (exp. 5) 26^{o}C	0.6555	-0.6591	0.6528	0.6581	-0.674	-0.6442
	$SrCO_3$ (exp. 6) 25^{o}C	0.6662	-0.6905	0.6633	0.6691	-0.6998	-0.6813
	$SrCO_3$ (exp. 7) 25^{o}C	0.6682	-0.7059	0.6652	0.6712	-0.7188	-0.6931
	$SrCO_3$ (exp. 8) 25^{o}C	0.6731	-0.7095	0.6676	0.6186	-0.7275	-0.6916
	Water 25^{o}C	0.6667	-0.7159	0.6618	0.6717	-0.7382	-0.6937
CUF 07	Low Solids	0.5716	-0.6039	0.5716	0.5813	-0.6241	-0.5838

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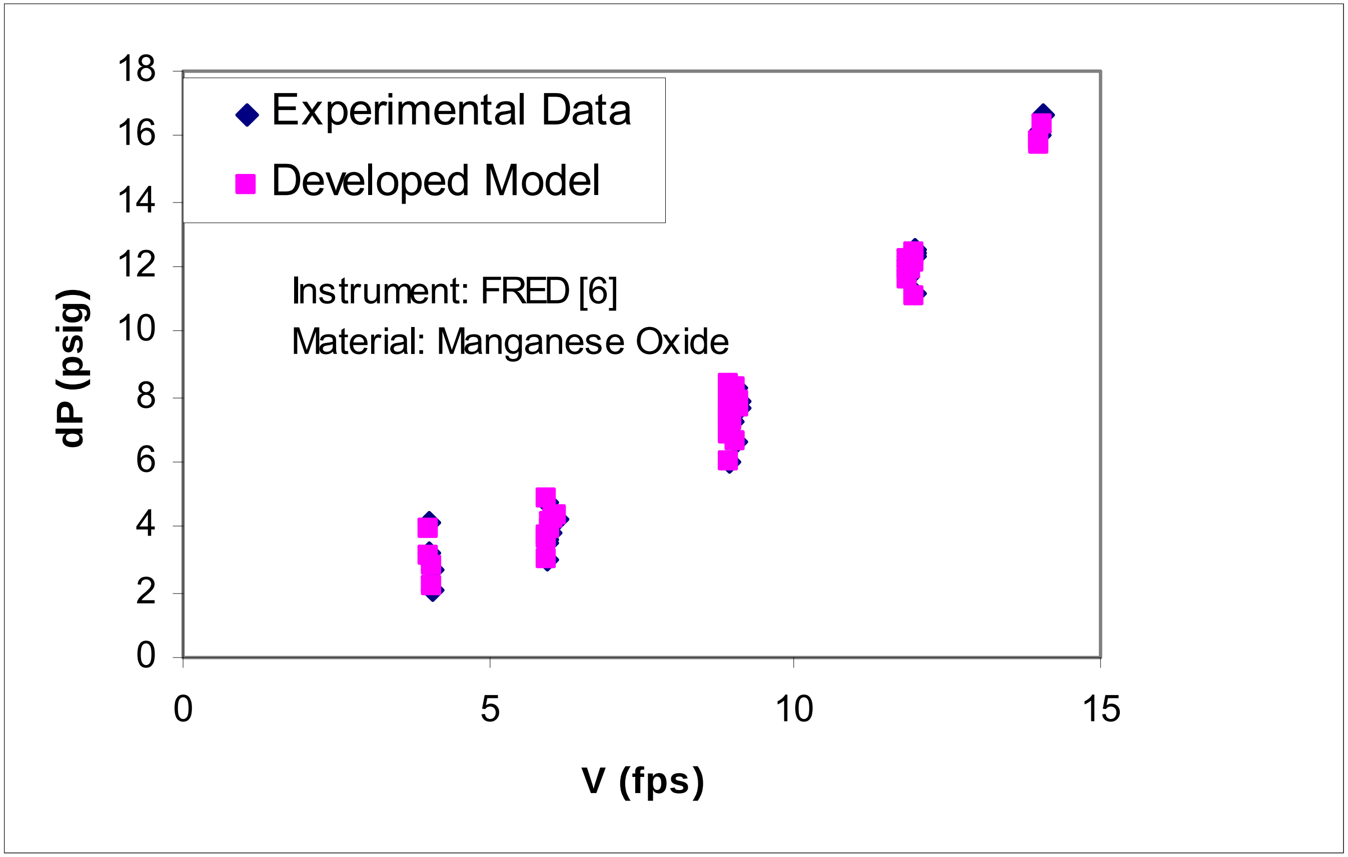
Results



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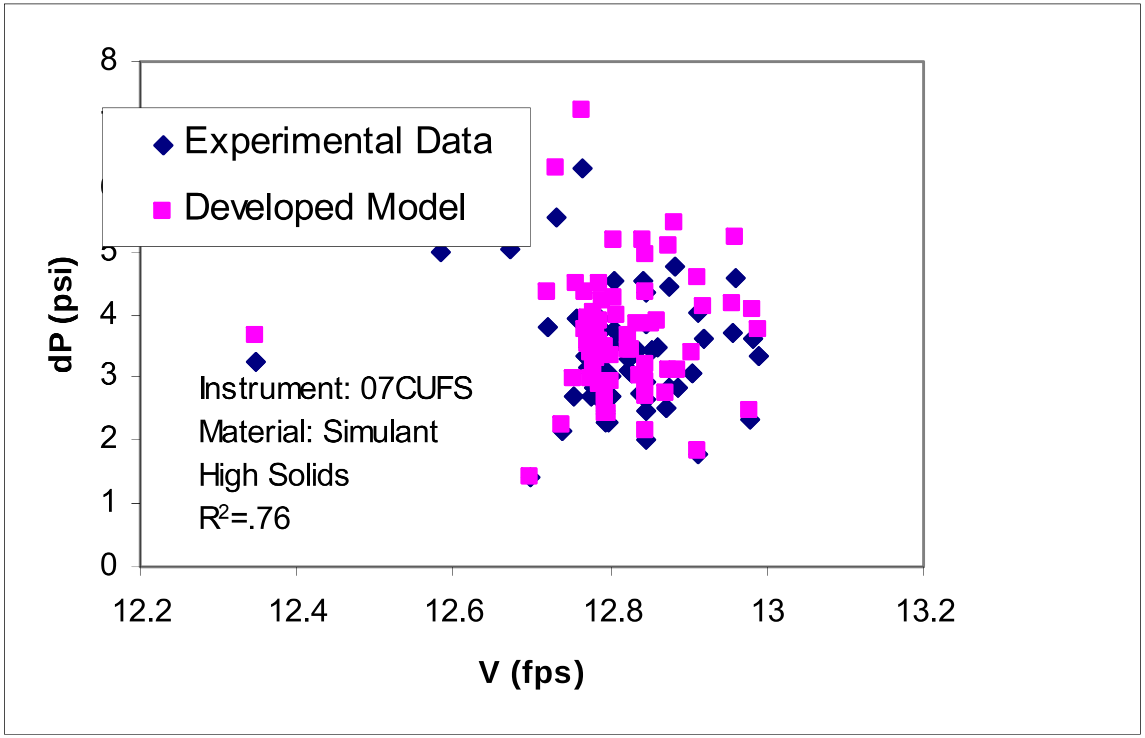
Results



Forecast - data was not utilized to fit parameters of model.

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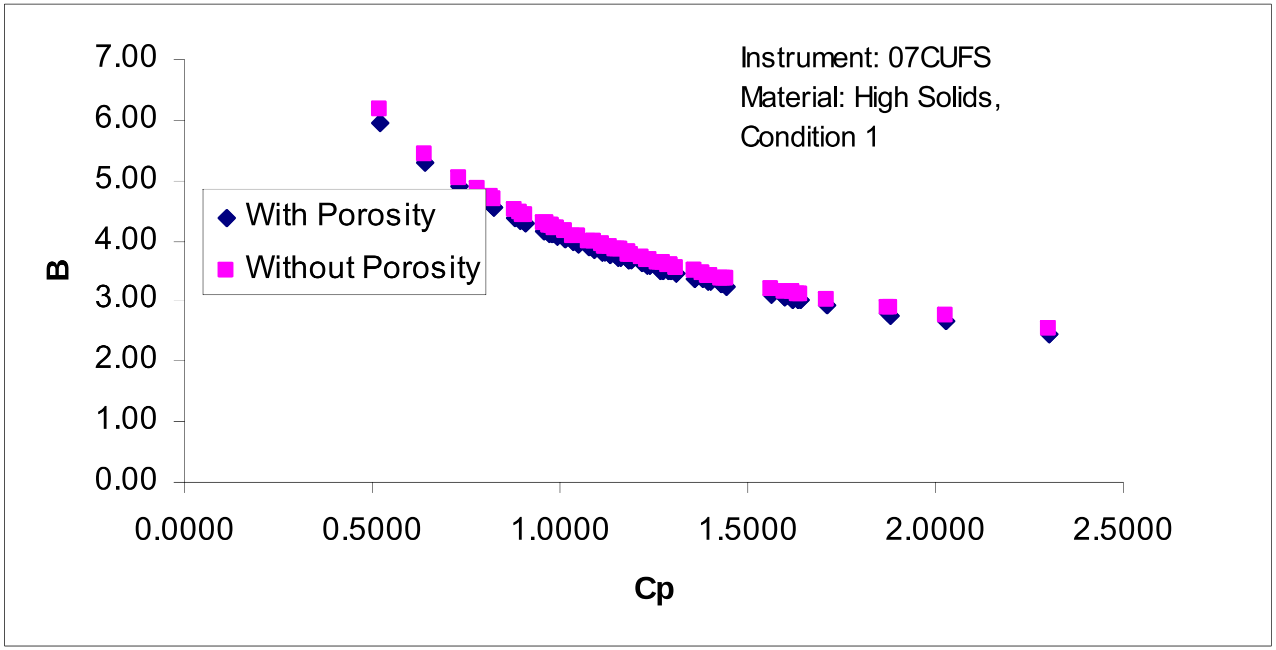
Results
(Porosity?)



$$B1 : B = a(C_p)^b (Q^*)^c$$

$$B2 : B = a(C_p)^b$$

$$Q^* = \frac{Q_s}{Q_p}$$

Qs = slurry discharge

Qp = permeate discharge

Another Approach

- Model of momentum
- Determination of velocity profiles
- Models for velocity profiles
- Determination of Boundary Layer
- Determination of rheological models
- Wall Shear
- friction factors

Momentum Integral

$$\frac{\tau_w}{\rho u_\infty{}^2} = \frac{u_0}{u_\infty} + \frac{\partial}{\partial x}\left[\delta \int_0^1 f(1 - f)\partial\eta\right]$$

Does not explicitly address pressure gradient.

Where τ_w is the wall shear, u_0 is the permeate flux, U_∞ bulk phase axial velocity, x is the direction down gradient, f is the velocity profile equation and is a function of □.

$$y = 0, \eta = 0, U_x = 0, f(\eta) = 0$$

$$y = \delta, \eta = 1, U_x = U_\infty, f(\eta) = 1$$

Measuring Velocity Profiles

- Hot Wire Anemometry
- Laser Doppler Velocimetry
- Laser PIV
  - Instantaneous 2-D map
  - Seed fluid
  - Visual
- Echo PIV
  - Instantaneous 2-D map
  - Seed fluid
  - Not Visual

Testing Methodology

- Measure clear tube with Laser PIV
- Measure clear tube with Echo PIV
- Measure sintered, porous tube with Echo PIV
- Material: iodine + silica solution (Newtonian)
- Material: iodine + silica + Xanthum gum (non-Newtonian)

Velocity Profile Models (Newtonian)

$$\frac{U_Z}{U_{Z,\max}} = 1 + a_1\left(\frac{r}{r_0}\right)^2 + a_2\left(\frac{r}{r_0}\right)^{2m}$$

$$a_1 = \frac{s-m}{m-1},\ a_2 = \frac{1-s}{m-1}$$

$$s = \frac{U^* r_0}{2\nu U_\infty}$$

The value of m will be determined as part of the research.

Brodkey (1966). Phenomena of Fluid Flows; John Wiley and Sons.

NSU/Case Western Reserve University

Velocity Profile Models (non-Newtonian)

$$\frac{u}{U\infty} = 1 + a_1\left(\frac{r}{r_0}\right)^{\frac{n+1}{n}} + a_2\left(\frac{r}{r_0}\right)^{2m}$$

$$a_1 = \frac{s - m}{m - (m+1)/2n}, a_2 = \frac{(n+1)/2n - s}{m - (n+1)/2n}$$

$$s = \frac{(y_0^{\ +})^{1/n}}{2U_0^{\ *}}$$

Determining s and m will be further refined as part of the research.

Brodkey (1966). Phenomena of Fluid Flows; John Wiley and Sons.

Research Questions

Research Questions that are being asked as part of this research include

- Can a reliable method be found for measuring velocity profiles?
- How does f change as C_b changes?
- How are the results of this study and a FaST Team study related?
- How do the parameters within velocity profile model change with C_b and what factors affect the change?
- Can a useful friction factor correlation be established?
- Is a clear delineation between Newtonian/non-Newtonian behavior seen?
- How are the results of this study related to the 'Body of Knowledge'?
- What are the likely rheological models for this system?

D/C

- Research Component (FaST Team)
  - Porosity has no effect
  - B model affected by physical scale
  - Developed model appears to have a higher goodness-of-fit over Blasius model
- Research Component (NSU/Case Western)
  - Haven’t done research yet!